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Research Letter

Limits of Faraday's law in isolated nanoscale magnetic systems

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Abstract

Classical electromagnetic laws such as Faraday's law are commonly interpreted within a continuum framework in which magnetic flux is treated as a statistically stable observable arising from large ensembles of magnetic dipoles. As magnetic systems approach the isolated nanoparticle limit, this continuum interpretation becomes progressively less robust because the magnetic response is increasingly governed by discrete localized sources rather than coarse-grained ensemble averaging. This work examines the implications of this continuum-to-discrete transition for the classical flux-based formulation of electromagnetic induction. While Maxwell's equations and the superposition principle remain fully valid at all length scales, the conventional interpretation of magnetic flux as a continuum observable loses operational robustness in the discrete-source regime. We show that electromagnetic induction is more naturally represented through the time-dependent vector potential generated directly by localized magnetic sources, providing a source-oriented description that remains fully consistent with classical electrodynamics. The implications for nanoscale magnetometry and isolated magnetic nanoparticles are discussed, identifying the transition from continuum field descriptions to discrete-source representations relevant to emerging nanoscale magnetic systems.

Introduction

Magnetic flux density **B** is a foundational quantity in both classical and quantum descriptions of magnetism. In classical electrodynamics, it is defined as the magnetic flux per unit area perpendicular to the field,

$$B = \frac{d\Phi_B}{dA_\perp}, \Phi_B = \int \mathbf{B} \cdot d\mathbf{A},$$

where **B** is the magnetic flux density vector (magnetic field), with units of tesla (T), describing the local magnitude and direction of the magnetic field. Φ_B is the magnetic flux through a surface, with units of weber (Wb), defined as the surface integral of the magnetic flux density over that surface. $d\mathbf{A}$ is an infinitesimal oriented area vector, whose magnitude dA equals the differential surface area and whose direction is given by the unit normal $\hat{\mathbf{n}}$ to the surface, i.e., $d\mathbf{A} = \hat{\mathbf{n}}\, dA$. $A_\perp$ is the area perpendicular to the magnetic field, and $dA_\perp$ denotes the differential area element normal to **B**. The subscript $\perp$ indicates projection onto the direction perpendicular to the field. The dot product $\mathbf{B} \cdot d\mathbf{A}$ therefore selects the component of the magnetic field normal to the surface element, ensuring that only the perpendicular field contributes to the magnetic flux.

The equation is treated as a continuous vector field whose magnitude represents the local intensity of the magnetic field and whose direction follows the orientation of magnetic force lines.[1–4] This representation, reinforced by the familiar visualization of smooth magnetic flux lines tangent to **B** at every point, has shaped physical intuition in magnetism for nearly two centuries.

The origin of this picture can be traced to Faraday's introduction of "lines of force" in the 1830s, in which magnetic influence was envisioned as propagating along continuous curvilinear paths through space.[2] Maxwell later formalized Faraday's qualitative insights into a self-consistent mathematical field theory, establishing **B** as a divergence-free field governed by

$$\nabla \cdot \mathbf{B} = 0, \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

where **E** is the electric field vector (units of Vm^{-1}), together with the remaining Maxwell equations.[3] The condition $\nabla \cdot \mathrm{B} = 0$, supported by all experiments to date, implies that magnetic flux lines neither originate nor terminate, reflecting the absence of magnetic monopoles. In macroscopic materials, where the fields of vast numbers of microscopic dipoles superpose and are effectively averaged over macroscopic length scales, this smooth-field description is both experimentally accurate and conceptually compelling.[1–4]

Magnetic flux density is mathematically well defined through Maxwell's equations for any source configuration, including a single magnetic dipole, and the resulting field is obtained through the superposition of individual source contributions. The present work does not question this mathematical definition. Instead, it examines the physical interpretation of magnetic flux density as a continuum observable. In such continuum descriptions, experimentally measured magnetic fields generally represent coarse-grained responses from large ensembles of magnetic moments and therefore exhibit statistically stable behavior because microscopic variations are effectively averaged over the measurement volume. As the number of contributing dipoles becomes limited, however, the measured magnetic response becomes increasingly dependent on the discrete-source configuration, measurement geometry, thermal fluctuations, and observation timescale. The continuum interpretation of magnetic flux density consequently becomes progressively less robust, even though the underlying magnetic field remains uniquely defined by classical electrodynamics. Throughout this work, the magnetic flux quantum Φ_0 is introduced only as an order-of-magnitude reference scale for evaluating the experimental observability of flux generated by finite localized magnetic sources, rather than as a flux-quantization constraint on ordinary magnetic materials.

Implicit in this historical formulation, however, is the assumption that the magnetic medium contains an effectively infinite ensemble of microscopic dipoles. Under such conditions, the highly localized and anisotropic fields generated by individual electron spins and orbital currents blend into a smooth vector field that can be meaningfully described by a continuous **B**.[5–9] This assumption begins to fail in systems composed of only a few dipoles or in engineered structures where dipole–dipole interactions are weak or deliberately suppressed.

Recent advances in nanofabrication and colloidal synthesis now allow magnetic nanoparticles—such as Fe_3O_4 nanoparticles—to be spatially separated.[10] In this discrete regime, the classical notion of a measurable magnetic flux density becomes ambiguous, because "flux per unit area" presupposes statistical averaging over many sources. When the number of dipoles approaches unity ($N \to 1$), this averaging ceases to exist, and the physical meaning of **B** as a smooth continuum field becomes questionable.[11]

Formally, Maxwell's equations remain valid at all length scales. A single magnetic dipole produces a field that is divergence-free everywhere except at the source, and $\nabla \cdot \mathbf{B} = 0$ is not violated. The difficulty lies instead in interpretation. Below a critical number of dipoles N_{crit}, statistical fluctuations dominate, and the experimentally accessible observables no longer correspond to the continuum quantities invoked in classical electromagnetism. In this limit, the smooth flux density required for the conventional application of Faraday's or Ampère's laws no longer exists as a measurable entity.[11]

For an isolated magnetic nanoparticle, the magnetic field generated by the finite ensemble of atomic magnetic moments remains well defined through Maxwell's equations and the principle of superposition. The issue considered here is instead the continuum interpretation of this field. In macroscopic magnetic materials, magnetic flux lines provide a useful representation of a smoothly varying, coarse-grained magnetic field. For spatially isolated nanoparticles containing a limited number of contributing magnetic moments, however, the local magnetic response becomes increasingly dependent on the discrete-source configuration, particle geometry, and measurement conditions. Consequently, the conventional flux-line representation and the associated continuum interpretation of magnetic flux become progressively less robust, even though the underlying magnetic field remains fully defined. Consequently, the conventional flux-line representation and the associated continuum interpretation of magnetic flux become progressively less robust, even though the underlying magnetic field remains fully defined. In this regime, the classical flux-based formulation of Faraday's law becomes operationally inadequate because the magnetic flux associated with an isolated nanoscale source no longer represents a statistically stable continuum observable. For example, a sub-critical, effectively fluxless dipole moving near a macroscopic conducting loop produces no experimentally measurable current, not because Maxwell–Faraday's equation fails, but because the flux contribution from an isolated discrete source is insufficient to generate an observable macroscopic response. Faraday's law in its integral form,

$$\mathcal{E} = -\frac{d\Phi_B}{dt},$$

remains a fundamental consequence of Maxwell's equations. However, its conventional macroscopic interpretation relies on the existence of a well-defined magnetic flux linking a conducting circuit. For an isolated nanoscale magnetic source, the magnetic field remains fully defined, but the associated flux contribution may become highly localized and dependent on the surface geometry, source position, and measurement scale. In this regime, the classical flux-based description of induction becomes operationally inadequate as a representation of the measurable response. For example, a sub-critical magnetic dipole whose flux contribution is below the macroscopic detection scale and moving near a conducting loop may produce no experimentally measurable current, not because Faraday's law fails, but because the induced response is beyond the regime where a continuum flux description provides a robust observable.

At the nanoscale, electromagnetic induction can alternatively be described through the time-dependent vector potential $\mathbf{A}(\mathbf{r}, t)$, which remains well defined and contributes to the electromotive response of atomic-scale or nanoscale loops according to[7–9]

$$\mathcal{E} = -\frac{d}{dt}\oint_{\text{loop}} \mathbf{A}(\mathbf{r}, t) \cdot d\mathbf{l}.$$

Here, $\mathcal{E}$ represents the electromotive force (EMF) induced around the closed loop, defined as the work done per unit charge in transporting a test charge once around the loop (units: volts). At nanoscale dimensions, the induction response can therefore be understood through the local electromagnetic coupling between the magnetic source and the circuit, rather than solely through a macroscopic magnetic flux threading the surface. Thus, while flux-line-based continuum interpretations of induction become less applicable for isolated discrete magnetic systems, the underlying electromagnetic interactions remain fully preserved.

Quantum mechanics introduces an additional layer of discreteness. In superconducting loops, magnetic flux is quantized in units of $\Phi_0 = h/2e$, as predicted by the London theory and confirmed experimentally by Deaver and Fairbank and by Doll and Näbauer.[5–7] These results underscore a fundamental tension between classical electromagnetism, which treats flux as continuous, and quantum systems, where flux appears in indivisible quanta. How a continuous flux density emerges statistically from a large ensemble of discrete dipoles—or conversely, how it disappears in the single-particle limit—remains insufficiently explored in both classical and quantum frameworks.

The central premise of this work is that **B** should be regarded not as a fundamental physical field, but as an emergent, statistical quantity. It exists operationally only when a sufficient number of dipoles produce a smooth, averaged magnetic field. Below N_{crit}, magnetism becomes intrinsically granular, dominated by localized dipole fields rather than by a coherent flux density. In this view, the breakdown of continuum magnetism does not signal a failure of Maxwell's equations, but rather a failure of the observables traditionally used to interpret them.

The present work builds upon our previous study of the statistical emergence of magnetic flux density from finite magnetic dipolar systems,[11] but addresses a different scientific question. Reference[11] established that magnetic flux density is an emergent continuum observable whose physical interpretation depends on statistical averaging over sufficiently large ensembles of magnetic dipoles and identified the regime in which this continuum description becomes progressively less robust. The present manuscript begins from this regime limitation and examines its direct implication for electromagnetic induction. Specifically, if magnetic flux is no longer a statistically stable continuum observable in discrete magnetic systems, then the conventional flux-based interpretation of Faraday's law becomes increasingly dependent on continuum assumptions. The analysis presented here shows that, while Maxwell's equations remain fully valid, electromagnetic induction in this regime is more naturally represented through the time-dependent vector potential generated by localized magnetic sources, providing an operationally robust description of induction without relying on a statistically stable continuum magnetic flux.

This statistical interpretation is consistent with the established descriptions of finite dipolar systems, superparamagnetism, and nanoparticle magnetization, where ensemble size plays a decisive role in magnetic behavior.[7–11] It also provides a natural framework for understanding modern single-spin and few-spin measurement techniques—including scanning SQUID magnetometry, STM-controlled spin systems, and NV-center-based magnetometry—which probe magnetic phenomena far below the classical continuum limit.[12–14] Together, these considerations motivate a reexamination of Faraday's and Ampère's laws in isolated and nanoscale systems, where classical flux-based intuition must be replaced by a discrete, interaction-based perspective.

It is important to distinguish between the mathematical definition of the magnetic field and its continuum interpretation. Throughout this work, Maxwell's equations and the superposition principle are assumed to remain fully valid. The magnetic field produced by a finite number of magnetic dipoles is therefore always well defined as the vector sum of the individual dipolar contributions. The issue considered here is different: in continuum electromagnetism, magnetic flux density is commonly interpreted as a coarse-grained macroscopic observable representing the collective response of many magnetic sources. As the number of contributing dipoles becomes small, this continuum interpretation becomes progressively less robust because the measured response depends increasingly on the discrete spatial distribution, temporal fluctuations, and measurement geometry of individual sources. The present work therefore examines the limits of the continuum interpretation of magnetic flux density, rather than the validity of Maxwell's equations or the superposition principle.

Magnetic flux density is mathematically well defined through Maxwell's equations for any source configuration, including a single magnetic dipole, and the resulting field is obtained through the superposition of individual source contributions. The present work does not question this mathematical definition. Instead, it examines the physical interpretation of magnetic flux density as a continuum observable. In macroscopic magnetic systems, measured magnetic fields naturally represent coarse-grained averages over enormous ensembles of magnetic moments and therefore exhibit statistically stable continuum behavior. As the number of contributing dipoles becomes limited, however, the measured magnetic response depends increasingly on the discrete-source configuration, measurement geometry, and temporal fluctuations. The continuum interpretation of magnetic flux density consequently becomes progressively less robust, even though the underlying magnetic field remains uniquely defined by classical electrodynamics. Throughout this work, the magnetic flux quantum Φ_0 is introduced only as an order-of-magnitude reference scale for evaluating the experimental observability of flux generated by finite localized magnetic sources, rather than as a flux-quantization constraint on ordinary magnetic materials.

Definition of isolated magnetic nanoparticles

To formalize the concept of an isolated, or "fluxless," magnetic nanoparticle, we introduce two quantitative criteria: (i) a critical particle size below which a continuous magnetic flux line cannot be sustained (Fig. 1), and (ii) a minimum interparticle separation above which dipole–dipole interactions are negligible (Fig. 2). Together, these criteria define the regime in which classical flux-based electromagnetic laws, including Faraday's law, lose operational applicability at the nanoscale.

Figure 1 illustrates the conceptual transition between continuum and discrete descriptions of magnetic behavior. The distinction shown in the figure is not based solely on the physical size of an individual particle, because even a nanoscale magnetic particle contains a large number of atomic magnetic moments and generates a well-defined magnetic field through the superposition of their contributions. Rather, the critical distinction is whether the magnetic response can be interpreted as a statistically stable continuum quantity. In the continuum regime [Fig. 1(a)], the

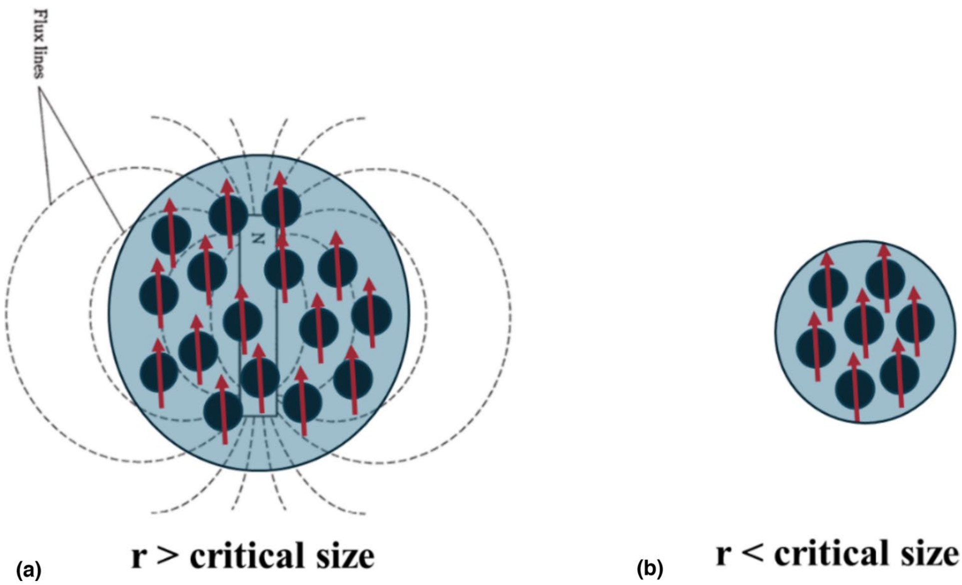


Figure 1. Schematic illustration of the transition from continuum to discrete magnetic-field descriptions. (a) A magnetic particle above the critical ensemble/continuum scale, where the collective contribution of aligned magnetic moments produces a smoothly varying magnetic field that can be represented by conventional continuous flux lines. (b) A finite nanoscale magnetic source below the critical scale for continuum interpretation. Although the particle still generates a well-defined magnetic field through the superposition of individual dipole contributions, the conventional continuous flux-line representation becomes progressively less robust because the magnetic response is dominated by discrete-source configuration rather than ensemble-averaged behavior. In both cases, the particle is assumed to be magnetically isolated, meaning that interparticle dipole–dipole interactions are negligible compared with thermal fluctuations ($U_{dd} \ll k_BT$). Such isolation can be achieved through sufficient particle separation, surface coatings, dispersion in a non-magnetic matrix, or low particle concentrations. Magnetic isolation eliminates collective dipolar coupling but does not by itself establish a continuum magnetic response, which requires a sufficiently large ensemble of magnetic moments for statistical averaging over the measurement volume.

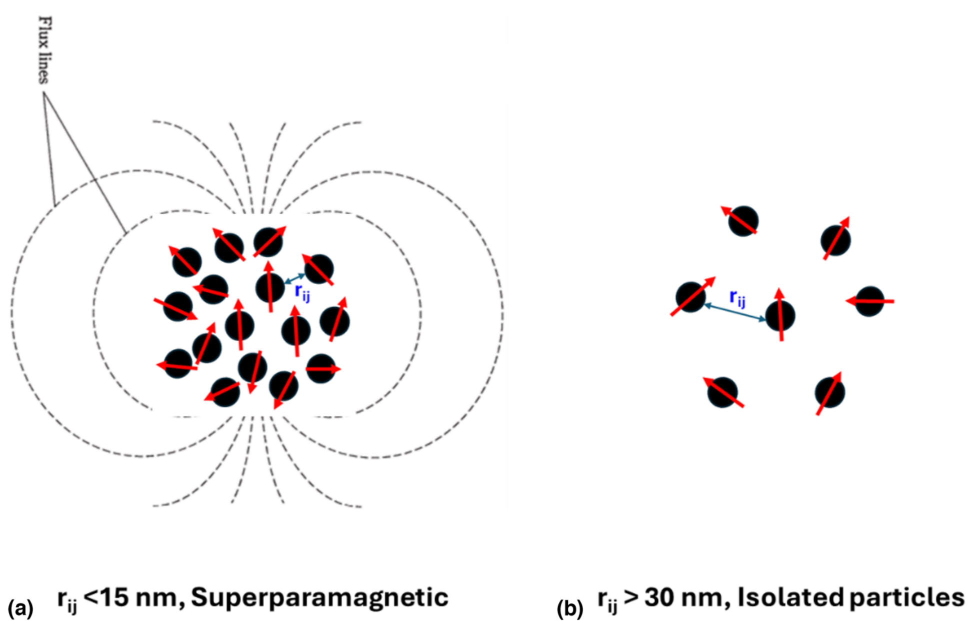


Figure 2. Schematic illustration of interparticle-spacing-dependent magnetic behavior. (a) Dense superparamagnetic assembly with interparticle spacing $r_{ij} < 15$ nm, where dipole–dipole interactions produce collective magnetic behavior and statistically stable continuum magnetic flux density. Under these conditions, continuous magnetic flux lines provide an appropriate macroscopic representation, and conventional flux-based electromagnetic descriptions apply. (b) Magnetically isolated nanoparticles with $r_{ij} > 30$ nm, where dipole–dipole interactions are negligible $(U_{dd} \ll k_BT)$. Each nanoparticle behaves as an independent magnetic source, and the continuum interpretation of magnetic flux density becomes progressively less robust because the ensemble no longer satisfies the statistical conditions required for coarse-grained averaging.

collective contribution of a sufficiently large ensemble of magnetic moments produces a smoothly varying magnetic field that can be represented by conventional flux lines and characterized by a well-defined magnetic flux density. In the discrete-source regime [Fig. 1(b)], a finite isolated magnetic source still produces a magnetic field, but the absence of sufficient ensemble averaging causes the measured response to depend increasingly on the discrete-source configuration, spatial geometry, and measurement conditions. Consequently, the conventional continuum interpretation of magnetic flux density and flux-line visualization becomes progressively less robust. The schematic therefore represents a transition in the physical interpretation of magnetic observables rather than the disappearance of magnetic fields or a limitation of Maxwell's equations.

Critical particle size and flux quantization

We model a magnetic nanoparticle as a uniformly magnetized sphere of radius a with saturation magnetization M_s. For a representative Fe_3O_4 nanoparticle of diameter $D = 10$ nm ($a = 5$ nm) and $M_s \approx 4.8 \times 10^5$ A/m,[11] the magnetic moment is

$$m = M_s V = M_s \frac{4}{3}\pi a^3 \approx 2.5 \times 10^{-19}\, Am2$$

At this scale, the classical notion of magnetic flux as a smooth, divergence-free field becomes ill-defined. In bulk systems, continuous flux lines emerge from the collective averaging of many dipoles. For isolated or few-dipole systems, this averaging collapses, and the magnetic field becomes intrinsically discrete and fluctuating.

A physically meaningful size criterion can be obtained by considering magnetic flux quantization. The minimum flux that can be sustained as a closed loop is one flux quantum,

$$\Phi_0 = \frac{h}{2e} \approx 2.07 \times 10^{-15}\,\text{Wb}.$$

Assuming the particle supports an approximately uniform internal magnetic flux density B, the minimum cross-sectional area required to sustain a single flux quantum is

$$A_{\min} = \frac{\Phi_0}{B}.$$

Treating this area as circular yields a critical radius

$$r_c = \sqrt{\frac{\Phi_0}{\pi B}}.$$

This expression highlights the intrinsic field dependence of the critical size. For representative values of B:

- $B = 1\,\text{T}$: $r_c \approx 25.7$ nm
- $B = 0.1\,\text{T}$: $r_c \approx 81.2$ nm
- $B = 0.01\,\text{T}$: $r_c \approx 256.7$ nm

These values indicate that as the internal magnetic field decreases, the minimum particle size required to sustain even a single quantum of magnetic flux increases rapidly, following

$$r_c \propto \frac{1}{\sqrt{B}}.$$

Particles with dimensions below this critical radius cannot sustain closed magnetic flux loops in a physically meaningful sense (Fig. 1). In this regime, the magnetic response is dominated by discrete dipole behavior rather than continuous field distributions, rendering classical flux-based descriptions—including Faraday induction—operationally invalid for isolated particles. Importantly, this breakdown does not imply a violation of Maxwell's equations, such as $\nabla \cdot \mathbf{B} = 0$. Rather, it reflects the collapse of the continuum approximation itself, which is statistical in origin and requires sufficiently large dipole ensembles to be meaningful.

The classical notion of magnetic flux requires averaging over many dipoles to produce a smooth, divergence-free fieldB .[1,2] Fluctuations in the magnetic field of N independent dipoles scale as

$$\frac{\delta B}{\langle B \rangle} \sim \frac{1}{\sqrt{N}} \Rightarrow N_{\text{crit}} \sim 1,$$

with practical tolerances requiring[11]

$N \gtrsim 100$for 10% relative stability, or$N \gtrsim 10^4$for 1% stability.

For a single nanoparticle, $N \lesssim N_{\text{crit}}$, and no smooth flux line exists, rendering conventional Faraday induction inapplicable. Table I summarizes the material parameters, magnetic energy scales, and numerical criteria introduced in this section that define the isolated (fluxless) nanoparticle regime at room temperature.

Critical interparticle spacing

Dipole–dipole interactions can reintroduce collective behavior if particles are sufficiently close. The maximum interaction energy for two aligned dipoles is

$$U_{dd}(r) = \frac{\mu_0}{4\pi} \frac{2m^2}{r^3}.$$

Comparing this interaction energy with the thermal energy at 300 K,

$$k_B T \approx 4.14 \times 10^{-21}\,\text{J},$$

the condition for negligible magnetic coupling is

$$U_{dd} \ll k_B T.$$

Solving $U_{dd}(r) = 0.1 k_B T$ yields the minimum center-to-center separation required for magnetic isolation,

$$r_{\text{iso}} \approx 31.2\text{nm},$$

corresponding to a surface-to-surface separation of approximately 21.2 nm for 10 nm Fe_3O_4 nanoparticles.

Table I. Numerical criteria for isolated (fluxless) 10 nm Fe3O4 nanoparticles at 300 K.[11]

Parameter	Symbol	Value	Unit
Particle radius	a	5	nm
Particle diameter	D	10	nm
Particle volume	V	5.24×10^{-25}	m^3
Saturation magnetization	M_S	4.8×10^5	A/m
Magnetic moment	m	2.51×10^{-19}	$A{\cdot}m^2$
Thermal energy	$k_B T$	4.14×10^{-21}	J
Dipole–dipole distance:$U_{dd} = k_B T$	r	14.5	nm
Dipole–dipole distance:$U_{dd} = 0.1 k_B T$	r	31.2	nm
Dipole–dipole distance:$U_{dd} = 0.01 k_B T$	r	67.3	nm
Critical number of dipoles for flux definition	N_{crit}	1	–
Practical number of dipoles for 10% stability	N	100	–
Practical number of dipoles for 1% stability	N	10^4	–

For comparison,

$$U_{dd} = k_B T \Rightarrow r \approx 14.5\text{nm},$$

$$U_{dd} = 0.01 k_B T \Rightarrow r \approx 67.3\text{nm}.$$

Particles separated by $r \gtrsim 30$ nm can therefore be regarded as effectively magnetically isolated under ambient conditions.

Definition of isolated nanoparticles

We define the discrete-source regime by two independent conditions:

1. Finite-source condition: the number of contributing magnetic particles satisfies

$$N < N_{\text{crit}},$$

such that a statistically stable continuum magnetic flux density no longer emerges.[11] The limiting case corresponds to a single magnetic nanoparticle ($N = 1$), representing the ultimate discrete magnetic source.

2. Magnetic-isolation condition: neighboring particles satisfy

$$r \gtrsim r_{\text{iso}} \approx 30\text{nm},$$

so that

$$U_{dd} \ll k_B T,$$

and dipole–dipole interactions are negligible.

Under these conditions, each nanoparticle behaves as an independent magnetic source. Its magnetic field remains uniquely determined by Maxwell's equations and the superposition principle.[2–4] The present work does not question the existence of this field. Rather, it examines the continuum interpretation of magnetic flux density as an operational observable. When the number of statistically independent magnetic sources falls below N_{crit}, the coarse-grained averaging that underlies continuum magnetic flux density is progressively lost.[11] Consequently, the conventional macroscopic flux formulation of Faraday's law,

$$\mathcal{E} = -\frac{d\Phi_B}{dt},$$

becomes operationally non-robust in the discrete-source regime, whereas the vector potential remains well defined,

$$\mathcal{E} = -\frac{d}{dt}\oint_{\text{loop}} \mathbf{A}(\mathbf{r}, t) \cdot d\mathbf{l}.$$

providing a natural description of electromagnetic induction in nanoscale systems containing finite localized magnetic sources.[2–8] The discrete-source regime therefore forms a conceptual bridge between continuum electrodynamics and single-particle magnetism, while remaining fully consistent with Maxwell's equations. Modern single-spin and nanoscale magnetic measurement techniques are capable of probing this regime directly.[5–7,13]

It is important to distinguish between a magnetically isolated nanoparticle ensemble and a truly isolated magnetic particle. In conventional nanoparticle magnetometry, "isolated" or "non-interacting" particles generally refer to ensembles in which interparticle dipolar interactions are sufficiently weak that individual particle moments fluctuate independently. Nevertheless, the measured response remains an ensemble average over a large number of magnetic particles. The isolated-particle limit considered here instead corresponds to a single magnetic nanoparticle acting as an individual magnetic source. For a 10 nm Fe_3O_4 nanoparticle, the calculated critical separation required to suppress dipole–dipole interactions is approximately 30 nm. A single nanoparticle measured in an idealized ultra-sensitive SQUID magnetometer provides a conceptual realization of this limiting case because no neighboring magnetic particles exist within the interaction range. This distinction is essential because eliminating interparticle coupling does not eliminate the statistical ensemble averaging that characterizes conventional nanoparticle measurements.

The statistical framework developed here should be distinguished from the classical Stoner–Wohlfarth description of single-domain particles and its thermally activated extensions.[15,16] The Stoner–Wohlfarth model describes coherent magnetization reversal of individual single-domain particles under an applied magnetic field, while thermally activated models describe fluctuations of individual particle moments. In contrast, the present work addresses the statistical emergence of magnetic flux density as a continuum observable arising from an ensemble of localized magnetic sources.[11] For the 10 nm Fe_3O_4 nanoparticles considered here, the particles are treated as magnetically isolated superparamagnetic particles under the specified experimental conditions, such that interparticle dipole–dipole interactions are negligible. Thermal fluctuations therefore influence the orientation and ensemble-averaged magnetization of individual particles but are not the mechanism responsible for the continuum-to-discrete transition discussed in this work. Instead, that transition arises from the finite number of statistically independent magnetic sources contributing to the measured magnetic response.[11]

Experimental implications

The distinction between dense and isolated magnetic nanoparticles leads to clear and experimentally testable consequences for electromagnetic induction at the nanoscale. In dense assemblies of superparamagnetic nanoparticles, where the interparticle spacing is well below the magnetic isolation threshold and the effective number of contributing dipoles satisfies $N >> N_{crit}$, classical continuum assumptions are restored.[7–10] Collective averaging suppresses field fluctuations, yielding a smooth magnetic flux density B and a well-defined magnetic flux Φ_B through macroscopic pickup loops.[12–14] In this regime, Faraday's law applies in its conventional form, and the induced electromotive force scales with the time derivative of the total magnetic moment of the ensemble. Such behavior is routinely observed in magnetic particle imaging, inductive biosensing, and bulk magnetometry, where dense superparamagnetic nanoparticle suspensions generate readily measurable EMF signals.[10–14]

In contrast, for isolated nanoparticles satisfying the criteria defined above—namely, single-particle discreteness and negligible dipole–dipole coupling—the effective ensemble size remains below the critical threshold required for a smooth flux description.[11] The magnetic flux through any macroscopic loop fluctuates strongly in time and averages to nearly zero. As a result, the classical EMF predicted by $E = -d\Phi_B/dt$ becomes vanishingly small and experimentally undetectable in conventional induction coils. Any residual signal arises not from flux threading in the classical sense, but from local, time-dependent variations of the vector potential, which are only appreciable over nanoscale loops or in ultra-sensitive detection architectures.

This dichotomy suggests a direct experimental test. A controlled comparison can be performed by transporting dense and isolated Fe_3O_4 nanoparticles through the same superconducting quantum interference device (SQUID) pickup coil under identical conditions.[12–14] For dense nanoparticle clusters or agglomerates, a clear and reproducible SQUID signal is expected, consistent with classical flux induction.[5–9] For isolated nanoparticles dispersed well beyond the magnetic isolation distance, the induced signal should be strongly suppressed or absent, despite identical particle composition and magnetic moment.[11] The contrast between these two cases would directly demonstrate the breakdown of flux-based induction at the single-particle level.

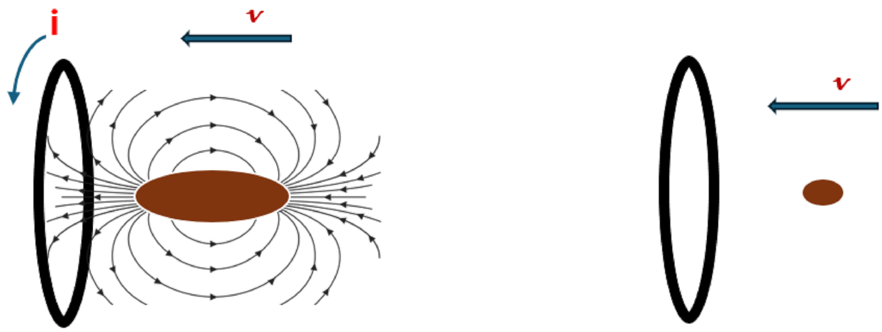
Further refinement of the experiment may be achieved by systematically tuning the interparticle spacing—for example, by diluting the nanoparticle concentration or embedding particles in a non-magnetic matrix with controlled separation. The resulting crossover from detectable classical EMF to a fluxless regime would provide quantitative validation of the proposed critical thresholds. Such experiments are well within the reach of existing SQUID magnetometry, nano-fabricated pickup loop technologies, and single-spin detection platforms, and would offer direct evidence that electromagnetic induction at the nanoscale is governed by discreteness and vector-potential dynamics rather than by classical magnetic flux alone.[11–14]

Faraday's law for an isolated nanoparticle

Faraday's law,

$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \Phi_B = \int \mathbf{B} \cdot d\mathbf{A},$$

expresses electromagnetic induction in terms of the temporal variation of magnetic flux threading a surface.[2] Within classical electrodynamics, the magnetic flux is always mathematically defined through the surface integral of the magnetic field, regardless of the source configuration. The present work does not question this definition. Instead, it examines the physical interpretation of the magnetic flux appearing in Faraday's law. In macroscopic magnetic systems, the measured magnetic flux represents a statistically stable continuum observable arising from the collective response of an enormous number of magnetic dipoles. Below the critical ensemble size N_{crit}, however, this continuum interpretation progressively loses its statistical robustness. The magnetic flux calculated from an isolated dipole remains formally well defined, but it becomes highly dependent on the instantaneous source position, dipole orientation, integration surface, and measurement geometry, rather than representing a reproducible coarse-grained quantity. Consequently, the limitation identified here does not arise because Φ_B or $d\Phi_B/dt$ vanish mathematically, but because the continuum magnetic flux appearing in the conventional macroscopic interpretation of Faraday's law no longer constitutes a statistically stable observable (Fig. 3). In this discrete-source regime, electromagnetic induction is therefore more naturally represented through the time-dependent vector potential generated



$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \quad \Phi_B = \int \mathbf{B}\cdot d\mathbf{A} \qquad\qquad \mathcal{E} = -\frac{d}{dt}\oint_{\text{loop}} \mathbf{A}(\mathbf{r},t)\cdot d\mathbf{l}.$$

(a) **Continuum (flux-sustaining) magnetic particle** (b) **Isolated 10 nm Fe_3O_4 nanoparticle**

Figure 3. Schematic comparison of electromagnetic induction in the continuum and discrete-source regimes. (a) A macroscopic magnetic body containing a statistically large ensemble of magnetic dipoles generates a smooth, continuum magnetic field that produces a statistically stable magnetic flux threading the conducting loop. The induced electromotive force is naturally described by the classical flux formulation of Faraday's law, $\mathcal{E} = -\frac{d\Phi_B}{dt}$. (b) An isolated 10 nm Fe_3O_4 nanoparticle generates a well-defined localized dipole field through Maxwell's equations and the superposition principle, but the magnetic flux threading the macroscopic loop no longer constitutes a statistically stable continuum observable because it depends sensitively on the instantaneous particle position, dipole orientation, loop geometry, and thermal fluctuations. In this discrete-source regime, electromagnetic induction is more naturally represented by the source-oriented vector-potential formulation $E = -\frac{d}{dt}\oint_{loop} A(r,t)\cdot dl$, which remains fully consistent with Maxwell's equations while avoiding reliance on a continuum magnetic flux description.

directly by the localized magnetic source, while remaining fully consistent with Maxwell's equations.

This conclusion holds irrespective of detector sensitivity: no current is induced in a conventional macroscopic loop because the smooth, ensemble-averaged flux assumed in classical Faraday's law does not exist for a single, isolated dipole. The stochastic fluctuations of the nanoparticle's magnetic field cannot produce a reproducible EMF through surface-integrated flux, and thus have no operational consequence for classical induction experiments.[11]

Magnetic detection techniques capable of probing isolated nanoparticles, such as nano-SQUIDs and NV-center magnetometry, operate in a fundamentally different regime.[12–14] These methods sense the local magnetic field or the time-dependent vector potential in the near field of the particle, rather than a macroscopic magnetic flux threading a surface. While such measurements can register the presence and dynamics of a single nanoparticle, they do not constitute classical Faraday induction and cannot be described by the flux-based formulation of the law.[1–9]

Thus, the statement "no current is induced" is physically meaningful and correct for isolated nanoparticles interacting with conventional conducting loops. Faraday's law is not violated—Maxwell's equations remain formally valid—but its operational applicability fails in the single-particle limit. Only when a sufficiently large ensemble of dipoles contributes to a smooth, statistically stable magnetic flux does the classical form of Faraday's law regain physical meaning.

Discussion

Classical electromagnetic laws versus discrete nanoscale systems

Classical electromagnetism is formulated as a continuum theory, in which its central quantities—magnetic flux density **B**, electric current density **J**, and magnetic flux Φ_B—are implicitly defined through spatial and temporal averaging over a large number of microscopic sources.[1,4] Although this assumption is rarely stated explicitly, it becomes critical when electromagnetic laws are applied at the nanoscale.

The magnetic flux density **B** is only operationally meaningful when it emerges from an ensemble of dipoles large enough to suppress statistical fluctuations. As shown above, relative field fluctuations scale as

$$\frac{\delta B}{\langle B\rangle} \sim N^{-1/2},$$

implying a critical ensemble size $N_{\text{crit}} \gg 1$ for a smooth, measurable field.[11] Below this threshold—in the extreme limit of a single magnetic nanoparticle or spin—**B** is no longer a well-defined continuum quantity, but a strongly fluctuating field whose instantaneous value depends on the stochastic orientation of the dipole moment.[11]

Faraday's law in its conventional form,

$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \Phi_B = \int \mathbf{B}\cdot d\mathbf{A},$$

therefore, presupposes the existence of a smooth and reproducible magnetic flux threading a surface.[1,2,4] For isolated dipoles, this requirement is not met. The magnetic flux through any finite surface either averages to nearly zero or exhibits fluctuations comparable to its mean value, rendering the classical electromotive force non-operational as a measurable quantity. In this sense, the apparent breakdown of Faraday's law at the nanoscale reflects not a failure of the underlying field equations, but a failure of the continuum interpretation imposed on intrinsically discrete systems.[11]

A similar limitation applies to Ampère-type formulations, which assume the existence of a continuous current density $\mathbf{J}(\mathbf{r})$.[1,4] At the level of a single magnetic dipole or a few localized spins, there is no continuous current distribution—only discrete microscopic currents or magnetic moments.[5–9] While the formal field solution for a point dipole exists, its interpretation in terms of macroscopic circulating currents and smoothly varying fields becomes physically ambiguous. Together, these considerations expose a conceptual gap between classical electromagnetic laws and nanoscale magnetic systems composed of only a few degrees of freedom.

Vector potential as the fundamental field in discrete regimes

In contrast to flux density and current density, the vector potential $\mathbf{A}(\mathbf{r}, t)$ remains well defined at all length scales. By definition,

$$\mathbf{B} = \nabla \times \mathbf{A}, \mathcal{E} = -\frac{d}{dt} \oint \mathbf{A} \cdot d\mathbf{l},$$

independent of whether the underlying sources form a continuum or a discrete set.[1,4] For time-dependent magnetic dipoles, the vector potential varies locally in space and time even when the notion of a global magnetic flux becomes ill posed.

This observation provides a natural generalization of Faraday's law for nanoscale systems. In the discrete regime, electromotive forces arise not from the time derivative of a smooth magnetic flux threading a surface, but from the temporal variation of the vector potential along a closed conducting path. The induced EMF is therefore governed by local field dynamics rather than by flux conservation arguments that rely on coarse-grained averaging.[11]

Importantly, this formulation permits the theoretical existence of extremely small but finite induced currents generated by single magnetic dipoles or isolated nanoparticles in nearby nanoscale loops. Such effects lie outside the scope of the classical flux-based picture, yet they are fully consistent with electromagnetic theory when expressed in terms of potentials. This perspective aligns naturally with modern experimental capabilities, including nano-SQUID magnetometry, single-spin detection, and NV-center-based sensing, which operate precisely in regimes where classical continuum assumptions no longer hold.[12–14]

Taken together, these results indicate that the vector potential, rather than the magnetic flux density, constitutes the more fundamental electromagnetic quantity in discrete nanoscale systems. The concept of isolated or fluxless magnetic nanoparticles thus serves not only as a physical classification, but also as a conceptual bridge between classical electromagnetism and the emerging physics of single-particle and single-spin phenomena at the nanoscale.[11]

Conclusion

In conclusion, this work demonstrates that the conventional continuum formulations of Faraday's and Ampère's laws are emergent macroscopic descriptions rather than universally applicable microscopic descriptions. Their physical validity relies on statistical averaging over large ensembles of magnetic dipoles. In isolated nanoscale magnetic systems, where this averaging no longer exists, the continuum concepts of magnetic flux and enclosed current cease to provide a physically meaningful description of electromagnetic induction. At the nanoscale, where magnetic systems may consist of a single particle or only a few sparsely distributed nanoparticles, the statistical averaging required for a smooth continuum magnetic flux density B and a reproducible magnetic flux Φ_B no longer exists. Consequently, the conventional flux-based description of electromagnetic induction ceases to be physically meaningful, although the underlying electromagnetic interactions governed by Maxwell's equations remain fully valid. We show that electromagnetic induction in this discrete regime is more naturally described in terms of the time-dependent vector potential, which remains well defined irrespective of ensemble size and provides a consistent framework for understanding nanoscale electromotive forces. The concept of isolated, or "fluxless," magnetic nanoparticles introduced here establishes a clear conceptual boundary between continuum and discrete electromagnetic behavior. By identifying the conditions under which classical electromagnetic observables emerge through statistical averaging—and conversely, when continuum concepts such as magnetic flux lose their physical meaning—this work bridges classical electromagnetism with the physics of isolated nanoscale magnetic systems. These findings provide a new perspective on the applicability of Faraday's law at the nanoscale and motivate the development of generalized electromagnetic descriptions tailored to discrete-source and quantum-limited systems.

Author contributions

D.S conceived the theoretical concept, performed the mathematical calculations, analyzed the data, and wrote the manuscript.

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Data availability

The data supporting the findings of this study are available in a publicly accessible repository.

Declarations

Conflict of interest

The author declares no conflict of interest.

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