Neuroscience Letters 626 (2016) 54–58

Contents lists available at ScienceDirect

Neuroscience Letters

journal homepage: www.elsevier.com/locate/neulet

Research paper

Fractal analysis reveals subclasses of neurons and suggests an explanation of their spontaneous activity

Luis H. Favela[a,b,*,1], Charles A. Coey[c,2], Edwin R. Griff[d,3], Michael J. Richardson[e,4]

[a] Department of Philosophy, University of Central Florida, Orlando, FL, United States
[b] Cognitive Sciences Program, University of Central Florida, Orlando, FL, United States
[c] Department of Psychology, College of the Holy Cross, Worcester, MA, United States
[d] Department of Biological Sciences, University of Cincinnati, Cincinnati, OH, United States
[e] Center for Cognition, Action, and Perception, Department of Psychology, University of Cincinnati, Cincinnati, OH, United States

HIGHLIGHTS

- Dynamics of single neurons in an anesthetized animal model were analyzed via fractal methods.
- Detrended fluctuation analysis (DFA), a kind of fractal analysis, was utilized to bolster the case for two subclasses of mitral cells.
- Fractal methods (DFA) captured differences in dynamics at longer time scales between two groups of neurons; a difference not revealed by standard statistical methods.
- One group of neurons demonstrated dynamics in the 1/*f* spectrum.
- Self-organized criticality is one explanation of dynamics that fall within the 1/*f* spectrum, and is thus a potential mechanism driving the spontaneous activity of particular classes of neurons.

ARTICLE INFO

Article history:
Received 12 March 2016
Received in revised form 6 May 2016
Accepted 10 May 2016
Available online 14 May 2016

Keywords:
Detrended fluctuation analysis
Fractal scaling
Mitral cells
Olfactory bulb
Self-organized criticality
Spontaneous activity

ABSTRACT

The present work used fractal time series analysis (detrended fluctuation analysis; DFA) to examine the spontaneous activity of single neurons in an anesthetized animal model, specifically, the mitral cells in the rat main olfactory bulb. DFA bolstered previous research in suggesting two subclasses of mitral cells. Although there was no difference in the fractal scaling of the interspike interval series at the shorter timescales, there was a significant difference at longer timescales. Neurons in Group B exhibited fractal, power-law scaled interspike intervals, whereas neurons in Group A exhibited random variation. These results raise questions about the role of these different cells within the olfactory bulb and potential explanations of their dynamics. Specifically, self-organized criticality has been proposed as an explanation of fractal scaling in many natural systems, including neural systems. However, this theory is based on certain assumptions that do not clearly hold in the case of spontaneous neural activity, which likely reflects intrinsic cell dynamics rather than activity driven by external stimulation. Moreover, it is unclear how self-organized criticality might account for the random dynamics observed in Group A, and how these random dynamics might serve some functional role when embedded in the typical activity of the olfactory bulb. These theoretical considerations provide direction for additional experimental work.

1. Introduction

It is easy to think that what matters most about neurons is how they respond to stimulation and whether or not they produce action

* Corresponding author at: Department of Philosophy, University of Central Florida, Orlando, Florida, United States.
E-mail addresses: luis.favela@ucf.edu (L.H. Favela), coeyca@mail.uc.edu (C.A. Coey), edwin.griff@uc.edu (E.R. Griff), michael.richardson@uc.edu (M.J. Richardson).
[1] Department of Philosophy, University of Central Florida, 4000 Central Florida Boulevard, Orlando, FL 32816-1352, United States.
[2] Department of Psychology, College of the Holy Cross, 1 College Street, Box 38A, Worcester, MA 01610, United States.
[3] Department of Biological Sciences, University of Cincinnati, 721A Rieveschl Hall, Cincinnati, OH 45221-0006, United States.
[4] Center for Cognition, Action, and Perception, Department of Psychology, University of Cincinnati, 4150B Edwards 1, Cincinnati, OH 45221-0376, United States.

http://dx.doi.org/10.1016/j.neulet.2016.05.017
0304-3940/Published by Elsevier Ireland Ltd.

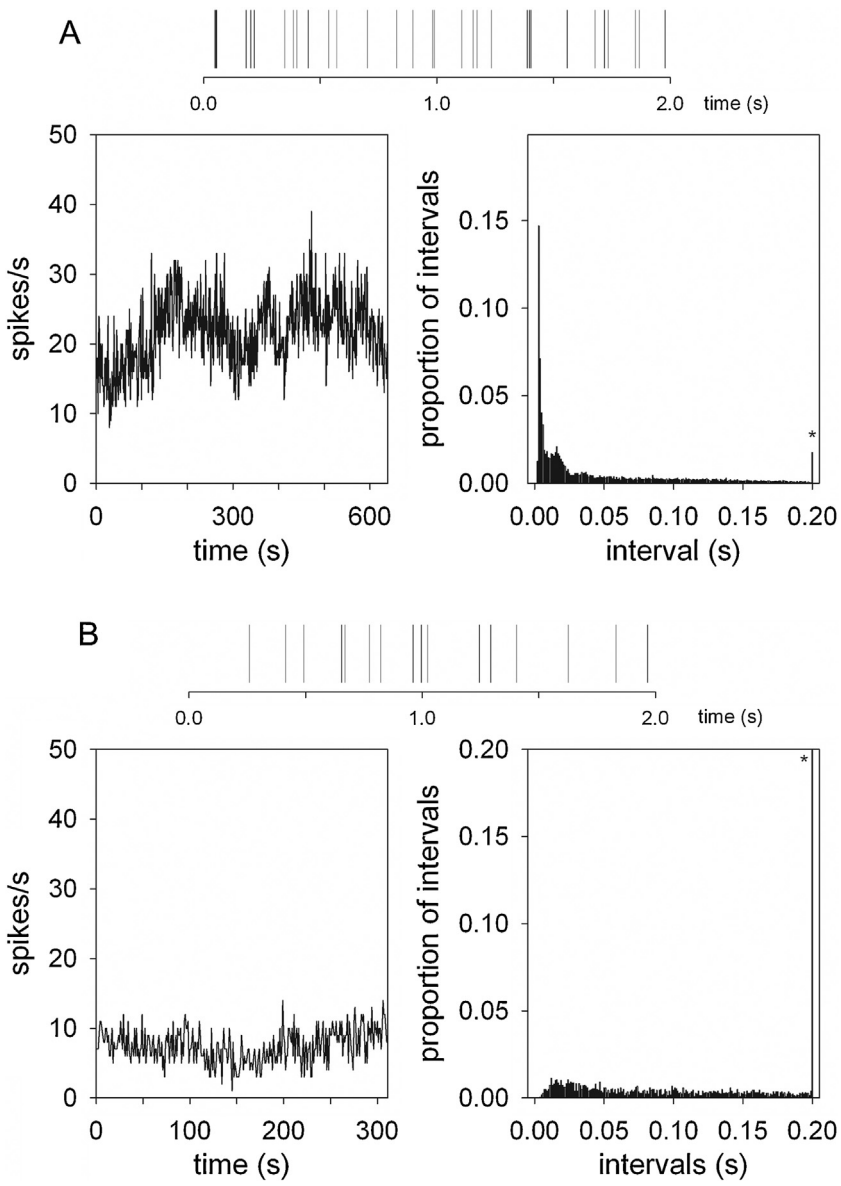

Fig. 1. Examples from recordings of the two classes of mitral cells, labeled A and B. At the top of each set of plots, is a short sample of spontaneous action potentials. The plots below on the left show the mean rate of activity in spikes/s. The plots on the right present interspike interval (ISI) histograms for each cell. Reproduced with permission from [45].

potentials. Such a conception suggests neurons function as simple on-off switches that change their state in response to inputs from other neurons. However, there is increasing evidence that such models (e.g., integrate-and-fire models) do not accurately describe real neurons [1,2]. Neurons are better understood as complex systems unto themselves that can exhibit dynamics that are nonlinear [3,4], stochastic [5], and subject to strong contextual effects [6]. As a complex system unto itself, a single neuron has its own dynamics separate from those evoked by external stimulation. These intrinsic dynamics can be referred to as the "spontaneous activity" of a neuron. Spontaneous activity in neurons has previously been considered unstructured and uncorrelated noise [7]. Contrary to this view, recent work has begun to provide evidence that spontaneous activity is not merely random noise [8], but is revealing of the inherent complex dynamics of neurons [9,10].

Research on the complexity of brain dynamics has tended to focus on how information propagates across large networks of neurons. Specifically, several studies have suggested that the brain tends to be organized near a critical state, wherein activity spreads in coherent "avalanches" of activity [11–14]. Theoretically, this organization is optimal for information transmission and processing, as the activity neither expires quickly nor spreads too widely across the network [15,16]. There are several biologically-plausible models of how the brain might *self-organize* to this critical state [17,18] without intervention by external forces or agents, involving only relatively simple rewiring rules, changes in synaptic efficacy, or a balance of excitatory and inhibitory connections [19–23]. Importantly, networks in this critical state show fractal dynamics, in which the avalanches have no one characteristic scale, but instead obey power-law scaling relations [24,25]. 'Fractal scaling' refers to a self-similar, scale invariant relationship between the fluctuation sizes in a system (i.e., avalanches) and how often fluctuations of that size occur, so that fluctuations across all scales show the same basic pattern (i.e., size-frequency relationship) [25–27].

This kind of fractal scaling, sometimes called "pink noise", has been discovered in a range of behavioral, physical, and physiological data, including earthquakes [28], heartbeats [29], and human reaction times [30]. In nature, such scaling appears to be more the

rule than the exception [31]. In neurological signals, fractal scaling seems to pervade every level of organization, from ion channel behavior to whole brain activity [32–36]. At the level of individual neurons, fractal scaling exists in the spike rate or interspike interval series, with fluctuations in the firing rate of the neuron obeying a power-law relation. However, it is true that such systems do not display fractal scaling at all times, and that deviations from fractal scaling can have important implications concerning the function and health of the system [37,38]. For instance, fractal analyses can reveal differences in heartbeat dynamics of people with congestive heart failure [39], reaction times during conditions of increased uncertainty [40], sex differences in brain matter and dyslexia [41,42], and brain activity during different sleep cycle phases [43]. Thus, studying such deviations can provide insight into the relationship between the functional organization of the system and the resulting dynamics [30].

The present work applies fractal analysis to the spontaneous activity of single neurons in an anesthetized animal model; specifically, mitral cells located in the rat main olfactory bulb. Previous research suggests that spontaneous activity could be inherent to the cells themselves, rather than driven by centrifugal inputs to the bulb or by activity in receptor neurons [44]. Other research has provided physiological evidence for the existence of two classes of mitral cells [45], suggesting that different kinds of cells might serve different functional roles within the bulb. This past work identified these two classes of cells based on summary statistics of the variability in their spiking activity, though they might also be distinguished by their dynamics. Even to the untrained eye, the time series and distributions displayed in Fig. 1 suggest different patterns of variation. We submitted the data from this earlier study [45] to fractal analysis, both to bolster the case for two subclasses of mitral cells and to point the way to an account of the intrinsic dynamics that drive their spontaneous activity.

2. Materials and methods

We utilized detrended fluctuation analysis (DFA) [34] to analyze a previously published data set concerning the spontaneous activity of two different subclasses of mitral cells in the main olfactory blub of rats [45]. This data set had previously been analyzed by an examination of the standard linear statistics of the single cell recordings (e.g., mean spike rate). A hierarchal cluster analysis of these statistics suggested the existence of two different subtypes of mitral cells. The present analysis determined whether these groups could also be distinguished in terms of the dynamical structure of their activity.

The details of the data collection can be found in the original article [45]. In short, spontaneous single-unit activity was recorded from mitral cells recorded in vivo from the main olfactory bulb of freely breathing anesthetized rats. The anesthetic plane was adjusted and maintained such that a toe pinch desynchronized the EEG without causing limb withdrawal. Mitral cells were identified before each recording using the depth where the electrically-evoked field potential from the lateral olfactory tract reversed polarity [46], the ability to drive the cell antidromically from the posterior piriform cortex, and histological confirmation of the electrode tip's position (dye spot) in the mitral cell body layer.

Spontaneous activity varied across units from 4.6 to 35.0 spikes/s, the latency to generate an antidromic action potential from 2.0 to 11.4 ms, and the coefficient of variation (CV) in interspike interval from 0.87 to 7.11 ms. The mean spontaneous activity for all mitral cell recordings was 17.38 ± 8.3 spikes/s. Hierarchical cluster analysis showed at least two subclasses of mitral cells. Post-hoc MANOVA indicated significant differences between the two groups based on mean rate, latency, and CV in interspike interval (Wilks' $\lambda = 0.1741$, $F[3,25] = 40.47$, $p < 0.001$). Examination of post-hoc univariate tests showed the groups differed significantly only in mean rate ($F[1,27] = 106.91$, $p < 0.01$).

The spontaneous activity of 29 cells was recorded for an average of 540 s (range 100–1243 s), yielding a "spike time" series for each cell. The original analysis suggested that 8 of these cells fell into one category (Group A) and 21 into another (Group B). As a first step in the present analysis, it was necessary to determine what series to submit to fractal analysis. The raw series of spike times could be collated into either an interspike interval (ISI) series or into a spike rate series, quantifying the number of spikes per unit time (1 s). In fact, the original article presented the data in both ways [45]. We chose the interspike interval series for two reasons. First, ISI was more sensitive in terms of reflecting variation in the cells' behavior. Second, the ISI series were much longer, which is preferable for fractal analyses. From the original data set we selected recordings that provided an ISI series of at least 4096 observations, and this led to a final sample of 14 series, with 4 and 10 series from Groups A and B, respectively. Although we do provide a cursory explanation of the DFA technique below, detailed description of these analyses is beyond the scope of this paper [25,47,48].

2.1. Fractal scaling in interspike interval series

DFA is one method for analyzing the fractal scaling of time series [49]. DFA works by assessing a scaling relation between the amount of variation in the series around local linear trends and the associated timescale (i.e., number of samples). The output of DFA is the Hurst exponent (H), where $H \approx 0.5$ indicates random, uncorrelated variation ("white noise"), $H \approx 1.0$ indicates fractal, power-law scaling ("pink noise"), and $H \approx 1.5$ indicates of Brownian motion ("brown noise") [47]. Hurst exponents can serve as indicators of system complexity [50], where 'complexity' refers to degrees of interaction among spatial and temporal aspects of a system. Exponents in the lower ($H \approx 0.5$) and higher ranges ($H \approx 1.5$) suggest the behavior results from an organization defined by weak interactions between system components, while exponents near $H \approx 1$ indicate an organization defined by the relatively strong interactions that lead to critical state behavior [51]. Additionally, DFA expects series of fractional Brownian motion (fBm) type, and thus series of a fractional Gaussian noise (fGn) type should be integrated prior to DFA [26,27]. Preliminary analyses suggested the current ISI series were fGn type series, and so they were integrated before DFA. Second, given the length of the ISI series (4096 points), the minimum and maximum window sizes were 8 and 1024 points, respectively. Finally, we used overlapping windows (50%) to provide better estimates of variability for each window size.

3. Results

Examination of the DFA plots (Fig. 2) suggested two scaling regions: one for shorter timescale, smaller windows (i.e., 8–64 points), and one for longer timescale, larger windows (i.e., 128–1024 points). For each ISI series, we calculated Hurst exponents for both scaling regions. A 2 (group) × 2 (scaling region) mixed ANOVA on these Hurst exponents suggested a significant interaction effect ($F[1,12] = 4.86$, $p = 0.048$). There was no difference in the shorter timescale scaling of Group A ($M = 0.38$, $SD = 0.09$) and Group B ($M = 0.43$, $SD = 0.06$), with cells in both groups falling near the region of random variation ($H \approx 0.5$). However, there was a significant difference between the groups at the longer timescale ($t[12] = -2.8$, $p = 0.016$), with cells in Group B ($M = 0.98$, $SD = 0.27$) showing much "pinker" variation than cells in Group A ($M = 0.57$, $SD = 0.14$). This pattern of results is shown in the average DFA plots in Fig. 2.

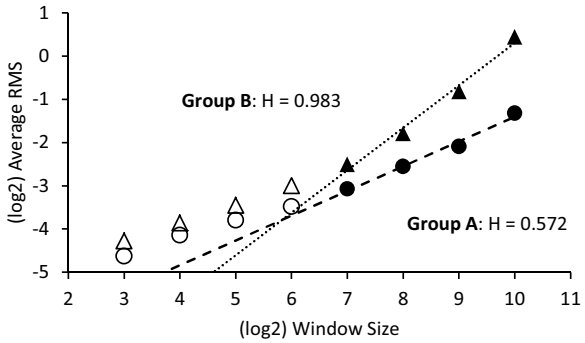

Fig. 2. Averaged DFA plots for ISI series from Group A (circles) and Group B (triangles) at both the shorter (empty) and longer timescales (filled).

As the sample size and dynamics of both groups were small, we conducted additional tests to verify the above effects. Specifically, we tested the four ISI series from Group A against sets of four series from Group B. Of the 210 unique sets from Group B, over half (52.4%) showed statistically higher longer-timescale exponents at the 0.05 level, and the majority (81.4%) was statistically higher at the 0.10 level. These tests also revealed generally good effect sizes; with the majority (62.9%) yielding a Cohen's *d* between 1.0 and 1.5. Although these tests are not entirely conclusive, they do suggest the difference between the groups is reliable within the present sample of cells. Hence, these results suggest that the longer timescale fluctuations in ISI for cells in Group A are far more random than the fractal, power-law scaling evident in the cells from Group B.

4. Discussion and conclusion

We applied fractal analysis to the spontaneous activity of neurons in the rat main olfactory bulb with the goal of providing further evidence of subclasses of mitral cells [45] and of gaining understanding about the processes governing spontaneous activity. The analysis confirmed that the two groups differed in spiking dynamics. One group of cells (Group B) revealed dynamics that fell within a pink noise, scaling region (i.e., $H \approx 1.0$), but the other did not (Group A). However, the analysis spawned a new question: What can account for the fractal dynamics exhibited in spontaneous activity in Group B neurons, and why is this fractal scaling absent in Group A neurons?

As mentioned above, past research on complexity in brain dynamics has suggested that power-law scaling is indicative of *self-organized criticality* (SOC) [14,52]. Again, the basic idea is that neural systems, at all different scales, tend to organize near critical states. When a system exhibits SOC, it is ordered enough to maintain spatial/temporal structure, but disordered enough to be adaptable to varying dynamics [52–54], which promotes optimal information flow. Though finding fractal scaling in the Group B neurons is consistent with SOC, we are left with two major questions. First, because the fractal scaling we found is in the *spontaneous activity* of the mitral cells, how do existing models of SOC apply to cases of minimal connectivity or truly isolated neurons? Second, how does the more random spiking dynamics of Group A neurons relate to these cells' function within the olfactory bulb?

Regarding the first question, SOC accounts of neural activity tend to focus on large networks of neurons [19–23]. In these models, the critical state behavior of the entire network, or "order parameter," is generally thought to emerge from "control parameters" defined at the scale of interacting neurons; such as synaptic efficacy. Adjusting these control parameters can move the system between subcritical, critical, and supercritical states. Gal and Marom [55] developed a dynamical mean-field-like model of SOC in single neuron activity:

$$\dot{x} = -f(y)x + g(x)(1 - x),$$

where the excitability of the neuron is the order parameter $\dot{x}$ and the stimulation rate is the control parameter (γ). Such models are inappropriate for the current data set. Although cells in the current data set were connected to other cells, they did not receive external stimulation. This raises an important issue, as SOC rests on an assumption of two timescales of activity: an external driving process and an internal relaxation process [53,54]. In the case of single neuron activity, it is possible that incoming synaptic stimulation functions as the external driving process for a neuron's action potentials. Stimulation builds up gradually and continuously and, once it reaches a threshold level, yields a spike that disperses energy in a rapid, discrete burst. In the case of spontaneous activity, the cell has been isolated from input from other neurons, and so this driving process is not tenable. Currently, it is unclear what other driving process might be able to meet the assumptions of SOC.

One alternative involves centrifugal input to the olfactory bulb. Several studies have implicated centrifugal inputs to the main olfactory bulb in olfactory behaviors. For example, cholinergic input to the olfactory bulb from the horizontal limb of the diagonal band influences odor discrimination and olfactory perceptual learning and memory [56,57]. Previous research indicates that the mean rate of spontaneous activity in the olfactory bulb was not significantly affected by blocking input from the olfactory epithelium in the nose [44]. However, this was not the case for centrifugal input. Thus, it is possible that the spontaneous activity of mitral cells is not entirely inherent to the cells, but is instead driven by centrifugal input. Another alternative implicates the ion channel dynamics. If the spontaneous activity is inherent to the cell, it might be driven by the intrinsic dynamics of the ion channels, which in principle can obey power-law scaling relations [33]. We suggest additional experiments blocking centrifugal input to the main olfactory bulb via the lateral olfactory tract. These experiments could determine how blocking centrifugal input affects spontaneous activity in mitral neurons, and would further clarify what processes drive this activity.

In regards to the second question, discussions of SOC are often accompanied by a discussion of "universality classes" [37,58]. A universality class refers to system-level behaviors that follow identical quantitative rules, despite being comprised of different material elements [58]. As a potential universality class, SOC can be postulated as an account for the critical behavior and fractal scaling dynamics in natural, complex spatial and temporal systems comprised of very different material substrates [53]. In short, SOC suggests a universal form of system organization, and the pervasiveness of fractal scaling might reflect the adaptive nature of this organization [31,37]. What remains unclear is whether deviations from fractal scaling truly implicate different functional organizations or universality classes [38]. If so, the random scaling observed in Group A cells could have deeper implications concerning the organization of the olfactory bulb. Why would the olfactory bulb be comprised of neurons with different scaling dynamics if, in fact, power-law scaling behavior is optimal for information processing? How might the dynamics of Group A cells differ during typical functioning when not blocked from their connections to receptor neurons?

The relationship between $1/f$ signals and SOC are already a contested issue, especially as relates to the structure and function of biological systems [30,59]. Notwithstanding such debates, the abundance of empirical evidence of fractal scaling in neural systems motivates the need for a theoretical framework to understand such experimental results. Universality classes such as SOC

may be needed to explain past and current experimental results and to point the way towards future research. Although, further theoretical and empirical work is needed to investigate the relation between neural activity and self-organized dynamics, we have demonstrated that fractal analysis can be a fruitful method when investigating neural dynamics and can point the way towards deeper theoretical understanding via universality classes such as self-organized criticality.

References

[1] A.V.M. Herz, T. Gollisch, C.K. Machens, D. Jaeger, Modeling single-neuron dynamics and computations: a balance of detail and abstraction, Science 314 (2006) 80–85.
[2] C. Koch, I. Segev, The role of single neurons in information processing, Nat. Neurosci. 3 (2000) 1171–1177.
[3] E.M. Izhikevich, Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting, MIT Press, Cambridge Massachusetts, 2007.
[4] B. Aguera y Arcas, A.L. Fairhall, W. Bialek, Computation in a single neuron: hodgkin and Huxley revisited, Neural Comput. 15 (2003) 1715–1749.
[5] W. Gerstner, W.M. Kistler, R. Naud, L. Paninski, Neuronal Dynamics: From Single Neurons to Networks and Models of Cognition, Cambridge University Press Cambridge, Massachusetts, 2014.
[6] T. Sharpee, N.C. Rust, W. Bialek, Analyzing neural responses to natural signals: maximally informative dimensions, Neural Comput. 16 (2004) 223–250.
[7] M.N. Shadlen, W.T. Newsome, The variable discharge of neurons: implications for connectivity, computation, and information coding, J. Neurosci. 18 (1998) 3870–3896.
[8] M. Raichle, The brain's dark energy, Science 314 (2006) 1249–1250.
[9] A. Mazzoni, F.D. Broccard, E. Garcia-Perez, P. Bonifazi, M.E. Ruaro, et al., On the dynamics of the spontaneous activity in neuronal networks, PLoS One 2 (2007) e439, http://dx.doi.org/10.1371/journal.pone.0000439.
[10] H. Hahn, T. Petermann, M.N. Havenith, S. Yu, W. Singer, D. Plenz, D. Nikolic, Neuronal avalanches in spontaneous activity in vivo, J. Neurophysiol. 104 (2010) 3312–3322.
[11] J.M. Beggs, D. Plenz, Neuronal avalanches in neocortical circuits, J. Neurosci. 23 (2003) 11167–11177.
[12] S. Bornholdt, T. Rohl, Self-organized critical neural networks, Phys. Rev. E 67 (2003), 066118-1–066118-5.
[13] N. Friedman, S. Ito, B.A.W. Brinkman, M. Shimono, R.E.L. DeVille, K.A. Dahmen, J.M. Beggs, T.C. Butler, Universal critical dynamics in high resolution neuronal avalanche data, Phys. Rev. Lett. 108 (2012) 208102-1–208102-5.
[14] J. Hesse, T. Gross, Self-organized criticality as a fundamental property of neural systems, Front. Syst. Neurosci. 8 (166) (2014), http://dx.doi.org/10.3389/fnsys.2014.00166.
[15] E. Bullmore, O. Sporns, The economy of brain network organization, Nat. Rev. Neurosci. 13 (2012) 336–349.
[16] W.L. Shew, D. Plenz, The functional benefits of criticality in the cortex, Neuroscientist 19 (2013) 88–100.
[17] J.A.S. Kelso, Dynamic Patterns: The Self-organization of Brain and Behavior, MIT Press, Cambridge Massachusetts, 1995.
[18] S.H. Strogatz, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering, Addison-Wesley Publishing, Reading, Massachusetts, 1994.
[19] A. Levina, J.M. Herrmann, T. Geisel, Dynamical synapses causing self-organized criticality in neural networks, Nat. Phys. 3 (2007) 857–860.
[20] F. Lombardi, H.J. Hermann, C. Perrone-Capano, D. Plenz, L. De Arcangelis, Balance between excitation and inhibition controls the temporal organization of neuronal avalanches, Phys. Rev. Lett. 108 (2012) 228703-1–228703-5.
[21] J. Peng, J.M. Beggs, Attaining and maintaining criticality in a neuronal network model, Phys. A 392 (2013) 1611–1620.
[22] M. Rubinov, O. Sporns, J.P. Thivierge, M. Breakspear, Neurobiologically realistic determinants of self-organized criticality in networks of spiking neurons, PLoS Comput. Biol. 7 (2011), e1002038-1–e1002038-14.
[23] M. Rybarsch, S. Bornholdt, Avalanches in self-organized critical neural networks: a minimal model for the neural SOC universality class, PLoS One 9 (2014), e93090-1–e93090-8.
[24] L.S. Liebovitch, Fractals and Chaos: Simplified for the Life Sciences, Oxford University Press New York, New York, 1998.
[25] J.G. Holden, Gauging the fractal dimension of response times from cognitive tasks, in: M.A. Riley, G.C. Van Orden (Eds.), Tutorials in Contemporary Nonlinear Methods for Behavioral Sciences, National Science Foundation, United States, 2005, pp. 267–318.
[26] A. Eke, P. Herman, J.B. Bassingthwaighte, G.M. Raymond, D.B. Percival, M. Cannon, et al., Physiological time series: distinguishing fractal noises from motions, Pflügers Arch. EJP 439 (2000) 403–415.
[27] D. Delignières, V. Marmelat, Theoretical and methodological issues in serial correlation analysis, in: M.J. Richardson, M.A. Riley, K. Shockley (Eds.), Progress in Motor Control: Neural, Computational, and Dynamic Approaches, NY: Springer, New York, 2013, pp. 127–148.
[28] P. Bak, C. Tang, Earthquakes as a self-organized critical phenomenon, J. Geophys. Res. 94 (1989) 15635–15637.
[29] C.-K. Peng, S. Havlin, J.M. Hausdorff, J.E. Mietus, H.E. Stanley, A.L. Goldberger, Fractal mechanisms and heart rate dynamics: long-range correlations and their breakdown with disease, J. Electrocardiol. 28 (1995) 59–65.
[30] G.C. Van Orden, J.G. Holden, M.T. Turvey, Human cognition and 1/f scaling, J. Exp. Psychol. Gen. 134 (2005) 117–123.
[31] B.J. West, M.F. Schlesinger, On the ubiquity of 1/f noise, Int. J. Mod. Phys. 3 (1989) 795–819.
[32] M.C. Teich, C. Heneghan, S.B. Lowen, T. Ozaki, E. Kaplan, Fractal character of the neural spike train in the visual system of the cat, J. Opt. Soc. Am. A 14 (1997) 529–546.
[33] S.B. Lowen, L.S. Liebovitch, J.A. White, Fractal ion-channel behavior generates fractal firing patterns in neuronal models, Phys. Rev. E 59 (1999) 5970–5980.
[34] R. Segev, M. Benveniste, E. Hulata, N. Cohen, A. Palevski, et al., Long term behavior of lithographically prepared in vitro neuronal networks, Phys. Rev. Lett. 88 (2002) 118102-1–118102-4.
[35] R. Hardstone, S.-S. Poil, G. Schiavone, R. Jansen, V.V. Nikulin, H.D. Mansvelder, K. Linkenkaer-Hansen, Detrended fluctuation analysis: a scale-free view on neuronal oscillations, Front. Physiol. 3 (2012), http://dx.doi.org/10.3389/fphys.2012.00450.
[36] J. Hu, J.-M. Lee, J. Gao, K.D. White, B. Crosson, Assessing a signal model and identifying brain activity from fMRI data by a detrending-based fractal analysis, Brain Struct. Funct. 212 (2008) 417–426.
[37] J.P. Sethna, K.A. Dahmen, C.R. Meyers, Crackling noise, Nature 410 (2001) 242–250.
[38] G.C. Van Orden, H. Kloos, S. Wallot, Living in the pink: intentionality, wellbeing, and complexity, in: C. Hooker (Ed.), Philosophy of Science 10: Philosophy of Complex Systems, Elsevier, 2016, pp. 639–683.
[39] A.L. Goldberger, C.K. Peng, L.A. Lipsitz, What is physiologic complexity and how does it change with aging and disease, Neurobiol. Aging 23 (2002) 23–26.
[40] C.T. Kello, B.C. Beltz, J.G. Holden, G.C. Van Orden, The emergent coordination of cognitive function, J. Exp. Psychol. Gen. 136 (2007) 551–568.
[41] K. Ahmadi, M. Ahmadlou, M. Rezazade, E. Azad-Marzabadi, F. Sajedi, Brain activity of women is more fractal than men, Neurosci. Lett. 535 (2013) 7–11.
[42] A.-L. Sandu, K. Specht, H. Beneventi, A. Lundervold, K. Hugdahl, Sex-differences in grey-white matter structure in normal-reading and dyslexic adolescents, Neurosci. Lett. 438 (2008) 80–84.
[43] V. Priesemann, M. Valderrama, M. Wibral, M. Le Van Quyen, Neuronal avalanches differ from wakefulness to deep sleep: evidence from intracranial depth recordings in humans, PLoS Comput. Biol. 9 (2013), e1002985-1–e1002985-14.
[44] J. Stakic, J.M. Suchanek, G.P. Ziegler, E.R. Griff, The source of spontaneous activity in the main olfactory bulb of the rat, PLoS One 6 (8) (2011), http://dx.doi.org/10.1371/journal.pone.0023990.
[45] R. Nica, S.F. Matter, E.R. Griff, Physiological evidence for two classes of mitral cells in the rat olfactory bulb, Brain Res. 1358 (2010) 81–88.
[46] W.T. Nickell, M.T. Shipley, Neurophysiology of the olfactory bulb, in: M.J. Serby, K.L. Chobor (Eds.), The Science of Olfaction, Springer-Verlag, Berlin, 1992, pp. 172–212.
[47] E.A. Ihlen, Introduction to multifractal detrended fluctuation analysis in Matlab, Front. Physiol. 3 (2012), http://dx.doi.org/10.3389/fphys.2012.00141.
[48] C. Brown, L. Liebovitch, Fractal Analysis, Sage, London, 2010.
[49] C.K. Peng, S.V. Buldyrev, S. Havlin, M. Simons, H.E. Stanley, A.L. Goldberger, Mosaic organization of DNA nucleotides, Phys. Rev. E 49 (1994) 1685–1689.
[50] J. Aguirre, R.L. Viana, M.A.F. Sanjuan, Fractal structures in nonlinear dynamics, Rev. Mod. Phys. 81 (2009) 333–386.
[51] J.G. Holden, G.C. Van Orden, M.T. Turvey, Dispersion of response times reveals cognitive dynamics, Psychol. Rev. 116 (2009) 318–342.
[52] P. Bak, C. Tang, K. Wiesenfeld, Self-organized criticality: an explanation of 1/f noise, Phys. Rev. Lett. 59 (1987) 381–384.
[53] P. Bak, C. Tang, K. Wiesenfeld, Self-organized criticality, Phys. Rev. A 38 (1988) 364–375.
[54] H.J. Jensen, Self-organized Criticality: Emergent Complex Behavior in Physical and Biological Systems, Cambridge University Press, Cambridge, Massachusetts, 1998.
[55] A. Gal, S. Marom, Self-organized criticality in single-neuron excitability, Phys. Rev. E 88 (2013) 06717-1–062717-5.
[56] N. Mandairon, C.J. Ferretti, C.M. Stack, D.B. Rubin, T.A. Cleland, C. Linster, Cholinergic modulation in the olfactory bulb influences spontaneous olfactory discrimination in adult rats, Eur. J. Neurosci. 24 (2006) 3234–3244.
[57] D.A. Wilson, M.L. Fletcher, R.M. Sullivan, Acetylcholine and olfactory perceptual learning, Learn. Mem. 11 (2004) 28–34.
[58] S. Yu, H. Yang, O. Shriki, D. Plenz, Critical exponents, universality class, and thermodynamic temperature of the brain, in: D. Plenz, E. Niebur (Eds.), Criticality in Neural Systems, Wiley-VCH Weinheim, Germany, 2014, pp. 319–333.
[59] E.-J. Wagenmakers, S. Farrell, R. Ratcliff, Human cognition and a pile of sand: a discussion on serial correlations and self-organized criticality, J. Exp. Psychol. Gen. 135 (2005) 108–116.